thrid point vectors

1. The diagram shows a square-based pyramid of height 8 units.

Square OABC has a side length of 6 units.

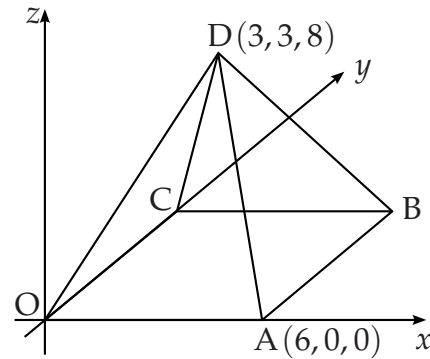The coordinates of A and D are $(6,0,0)$ and $(3,3,8)$.

C lies on the y-axis.

(*a*) Write down the coordinates of B. **1**

(*b*) Determine the components of $\overrightarrow{DA}$ and $\overrightarrow{DB}$. **2**

(*c*) Calculate the size of angle ADB. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	1	C	CN	G22	$(6,6,0)$	2002 P2 Q2
(*b*)	2	C	CN	G17	$\overrightarrow{DA} = \begin{pmatrix} 3 \\ -3 \\ -8 \end{pmatrix}$, $\overrightarrow{DB} = \begin{pmatrix} 3 \\ 3 \\ -8 \end{pmatrix}$	
(*c*)	4	C	CR	G28	$38 \cdot 7°$	

- $\bullet^1$ ic: interpret diagram
- $\bullet^2$ ic: write down components of a vector
- $\bullet^3$ ic: write down components of a vector

- $\bullet^4$ ss: use e.g. scalar product formula
- $\bullet^5$ pd: process lengths
- $\bullet^6$ pd: process scalar product
- $\bullet^7$ pd: process angle

- $\bullet^1$ B $= (6,6,0)$

- $\bullet^2$ $\overrightarrow{DA} = \begin{pmatrix} 3 \\ -3 \\ -8 \end{pmatrix}$

- $\bullet^3$ $\overrightarrow{DB} = \begin{pmatrix} 3 \\ 3 \\ -8 \end{pmatrix}$

- $\bullet^4$ $\cos A\widehat{D}B = \frac{\overrightarrow{DA}.\overrightarrow{DB}}{|\overrightarrow{DA}||\overrightarrow{DB}|}$
- $\bullet^5$ $|\overrightarrow{DA}| = \sqrt{82}, |\overrightarrow{DB}| = \sqrt{82}$
- $\bullet^6$ $\overrightarrow{DA}.\overrightarrow{DB} = 64$
- $\bullet^7$ $A\widehat{D}B = 38 \cdot 7°$

2. The diagram shows a cuboid OPQR,STUV relative to the coordinate axes.

P is the point $(4,0,0)$, Q is $(4,2,0)$ and U is $(4,2,3)$.

M is the midpoint of OR.

N is the point on UQ such that $UN = \frac{1}{3}UQ$.
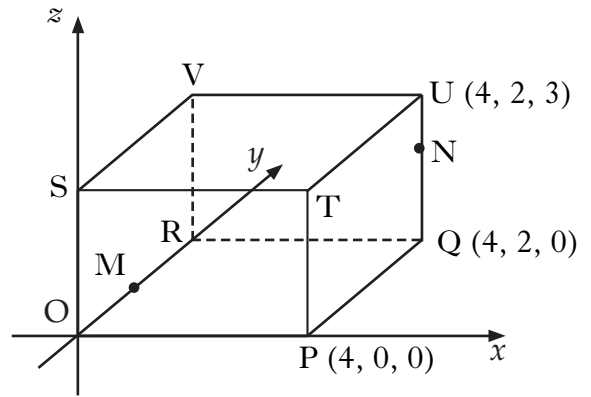
(a) State the coordinates of M and N. — **2**

(b) Express the vectors $\overrightarrow{VM}$ and $\overrightarrow{VN}$ in component form. — **2**

(c) Calculate the size of angle MVN. — **5**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	2	C	CN	G22, G25	$M(0,1,0), N(4,2,2)$	2010 P2 Q1
(b)	2	C	CN	G17	$\overrightarrow{VM} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}, \overrightarrow{VN} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$	
(c)	5	C	CN	G28	$76\cdot7°$ or $1\cdot339$ rad	

$\bullet^1$ ic: interpret midpoint for M
$\bullet^2$ ic: interpret ratio for N

$\bullet^3$ ic: interpret diagram
$\bullet^4$ pd: process vectors

$\bullet^5$ ss: know to use scalar product
$\bullet^6$ pd: find scalar product
$\bullet^7$ pd: find magnitude of a vector
$\bullet^8$ pd: find magnitude of a vector
$\bullet^9$ pd: evaluate angle

$\bullet^1$ $(0,1,0)$
$\bullet^2$ $(4,2,2)$

$\bullet^3$ $\overrightarrow{VM} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$

$\bullet^4$ $\overrightarrow{VN} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$

$\bullet^5$ $\cos M\widehat{V}N = \dfrac{\overrightarrow{VM}.\overrightarrow{VN}}{|\overrightarrow{VM}||\overrightarrow{VN}|}$

$\bullet^6$ $\overrightarrow{VM}.\overrightarrow{VN} = 3$
$\bullet^7$ $|\overrightarrow{VM}| = \sqrt{10}$
$\bullet^8$ $|\overrightarrow{VN}| = \sqrt{17}$
$\bullet^9$ $76\cdot7°$ or $1\cdot339$ rads or $85\cdot2$ grads

3. A box in the shape of a cuboid is designed with **circles** of different sizes on each face.
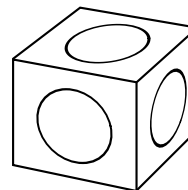
The diagram shows three of the circles, where the origin represents one of the corners of the cuboid. The centres of the circles are $A(6,0,7)$, $B(0,5,6)$ and $C(4,5,0)$.

Find the size of angle ABC.
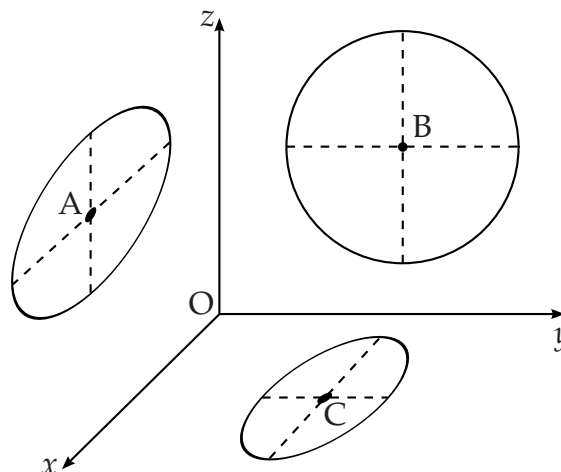
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Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	5	C	CR	G17, G16, G22		2001 P2 Q4
	2	A/B	CR	G26, G28	71·5°	

$\bullet^1$ ss: use $\frac{\overrightarrow{BA}.\overrightarrow{BC}}{|\overrightarrow{BA}||\overrightarrow{BC}|}$

$\bullet^2$ ic: state vector e.g. $\overrightarrow{BA}$

$\bullet^3$ ic: state a consistent vector e.g. $\overrightarrow{BC}$

$\bullet^4$ pd: process $|\overrightarrow{BA}|$

$\bullet^5$ pd: process $|\overrightarrow{BC}|$

$\bullet^6$ pd: process scalar product

$\bullet^7$ pd: find angle

$\bullet^1$ use $\frac{\overrightarrow{BA}.\overrightarrow{BC}}{|\overrightarrow{BA}||\overrightarrow{BC}|}$ *stated or implied by $\bullet^7$*

$\bullet^2$ $\overrightarrow{BA} = \begin{pmatrix} 6 \\ -5 \\ 1 \end{pmatrix}$

$\bullet^3$ $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 0 \\ -6 \end{pmatrix}$

$\bullet^4$ $|\overrightarrow{BA}| = \sqrt{62}$

$\bullet^5$ $|\overrightarrow{BC}| = \sqrt{52}$

$\bullet^6$ $\overrightarrow{BA}.\overrightarrow{BC} = 18$

$\bullet^7$ $A\hat{B}C = 71\cdot5°$

[END OF QUESTIONS]